Journal of Engineering Physics and Thermophysics, Vol. 71, No. 3, 1998

BREAKUP OF DROPS IN A PIPELINE

L. P. Pergushev and V. P. Tronov

UDC 532.529:622.276.8

We consider the problem of determining the size of drops in a turbulent flow of emulsion transported in a pipeline. A critical analysis of various formulas used for calculating the size of the drops is performed. Results of theoretical analysis and experimental data are used in a computer search for the best formula that would describe the breakup of water drops in an oil emulsion during its transport in a pipeline.

An analysis of the breakup of drops was made by Hinze on the basis of the hydrodynamic theory developed by Taylor and Kolmogorov. Thus, he emplasized the stochastic nature of the phenomenon, introduced the types of deformation of disintegrating drops, and showed their connection with the type of flow (uniform, accelerated, shear, rotational). He also obtained expressions for dimensionless numbers of the process of breakup [1]:

$$N_{\mathrm{We}} = \mu_c S/(\sigma/d)\,, \quad N_{vi} = \mu_d/\sqrt{\rho_d \sigma d}\,. \tag{1}$$

The breakup of drops occurs at N_{We} exceeding a critical value. The latter, in turn, depends on the ratio of the viscosities of the phases or, which is the same, on the value of N_{vi} and can vary in a wide range. Dispersion is difficult in the case of phases whose viscosities differ greatly.

For a highly turbulent flow in which the molecular viscosity of the continuous phase is much lower than that of the turbulent one and for small values of N_{vi}, Kolmogorov [2] and Hinze [1] independently obtained the following expression for the size of drops most stable to breakup:

$$d_* = C\,(\sigma/\rho)^{0.6}/\varepsilon^{0.4}\,, \tag{2}$$

where C is an empirical constant, which according to Hinze is $C = 0.725$. For a circular pipe $\varepsilon = \lambda u^3/(2D)$. With these relations taken into account, formula (2) can be rewritten as

$$d_*/D = 1.516\,\mathrm{Re}^{0.1}\,\mathrm{We}^{-0.6}\,, \tag{3}$$

where $\mathrm{Re} = \rho_c u D/\mu_c$; $\mathrm{We} = \rho_c u^2 D/\sigma$.

A different dependence was obtained by Sleicher [3]. Analyzing his own experimental data and data obtained by Clay and used by Hinze, he showed that d_* obeyed the following equation with 35% accuracy:

$$\frac{d_* \rho_c u^2}{\sigma}\sqrt{\left(\frac{\mu_c u}{\sigma}\right)} = 38\left[1 + 0.7\left(\frac{\mu_d u}{\sigma}\right)^{0.7}\right]. \tag{4}$$

He also noted that breakup occurs at the walls of the pipe, where turbulence is least isotropic and homogeneous. Here, predominantly two types of breakup are observed: in the first type, the drop is deformed and, when its length is four times its transverse dimension, it is halved; in the second type, fine drops are formed by shredding one large drop.

For water-in-oil emulsions the dimensionless combination $\mu_d u/\sigma$ is usually much less than unity, and Eq. (4) can be converted to a form similar to (3):

Tatar Research and Design Institute of Petroleum, Bugulma. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 71, No. 3, pp. 468-472, May-June, 1998. Original article submitted February 2, 1996.

$$d_*/D = 38\,\mathrm{Re}^{0.5}\,\mathrm{We}^{-1.5}. \tag{5}$$

In contrast to the Kolmogorov–Hinze theory (see formula (3)), in which viscous forces play virtually no role, their contribution to the size turns out to be very noticeable in relation (5). However, the size of the drops is more sensitive to the flow velocity. It is not difficult to see that according to Kolmogorov–Hinze $d_* \propto u^{-1.1}$ and according to Sleicher $d_* \propto u^{-2.5}$. This contradiction was successfully resolved by Rozentsvaig. He showed that in the Kolmogorov–Hinze theory account for the deformation of a drop due to the effect of the averaged-velocity gradient gives for the most stable size of a drop a dependence that is close to Sleicher's formula. The expression obtained by Rozentsvaig [4] has the form

$$\lambda\,(\rho_c d_* u^2/\sigma) = C_0\,(\sqrt{\lambda}\,D/d_*)^{-0.3}, \tag{6}$$

where the proportionality factor C_0 is a function of the viscosity ratio of the phases. By means of statistical processing of experimental data, Rozentsvaig obtained the following expressions for C_0:

$$C_0 = 4.27\,(\mu_d/\mu_c)^{-0.38}, \quad \mu_d/\mu_c < 1.05; \tag{7}$$

$$C_0 = 4.2, \quad 1.05 \le \mu_d/\mu_c < 2.40; \tag{8}$$

$$C_0 = 3.45\,(\mu_d/\mu_c)^{0.22}, \quad \mu_d/\mu_c > 2.40. \tag{9}$$

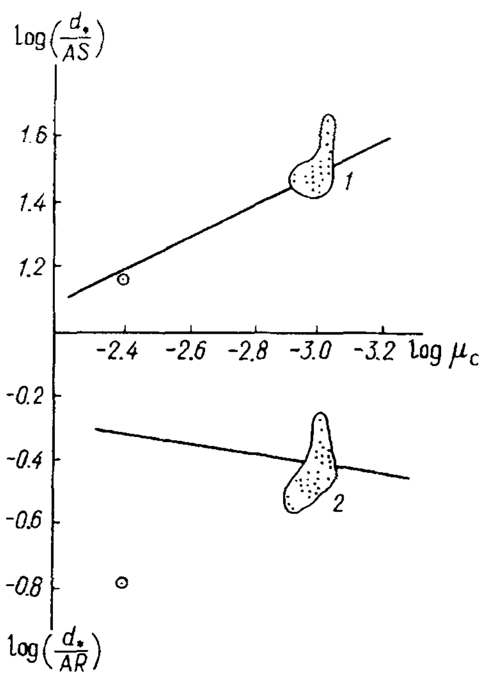
In application to pipelines and tubular apparatuses, the nonuniformity of the turbulent field near the walls leads to substantial enhancement of the role of viscous forces in the breakup of drops. From expressions (7)-(9) it follows that the effect will be maximum, i.e., the drops will turn out to be smallest, when the viscosities of the phases are nearly the same.

For water-in-oil emulsions it is necessary to use expression (7). Substituting it into Eq. (6) and solving for d_*/D gives the following formula

$$d_*/D = 52.68\,(\mu_d/\mu_c)^{-0.54}\,\mathrm{Re}^{0.41}\,\mathrm{We}^{-1.43}. \tag{10}$$

Comparing formula (10) with (5), we can actually see that the connection between d_*, We, and Re has virtually the same form. However, formula (10) is more general, since in contrast to (5), as well as to (3), it involves a factor that accounts for the difference between the viscosities of the phases. As regards numerical values, for a pipeline 0.2 m in diameter, oil whose density and viscosity are 870 kg/m^3 and 0.01 Pa·sec, respectively, and a phase tension of 0.02 N/m, the diameter of the water drops will be 3.48 mm according to formula (3) and 1.24 mm according to formula (5). For the same parameters formula (10) yields the values 1.34 and 4.64 mm for μ_d/μ_c equal to 1.0 and 0.1, respectively. The value 4.64 mm corresponds to a water-in-oil emulsion, and as μ_c increases, it will increase according to a power law with an exponent of 0.13 (see formula (10)). At the same time, according to formula (5), as the viscosity of the continuous phase increases, the diameter of the drops decreases according to a power law with an exponent of 0.5. Such contradictory behavior of the dependence of the diameter of the drops on the viscosity of the continuous medium results from the empirical nature of formulas (5) and (10). Thus, relations (7)-(9) were established for disperse systems the viscosity of whose phases varied within the limits $0.96 < \mu_c < 1.8$ mPa·sec, $0.5 < \mu_d < 32.1$ mPa·sec. In Sleicher's experiments the viscosity of the disperse phase had the same limits, whereas the range of variation of the viscosity of the continuous phase was wider and equalled (0.96–3.96) mPa·sec.

The goodness-of-fit of the relations considered to experimental data can be established most easily using Figs. 1 and 2.



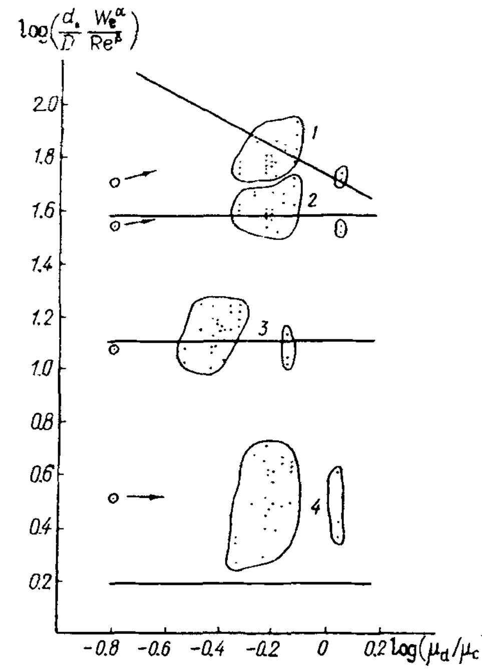


Fig. 1. Dependence of the size of the drops on the viscosity of the continuous phase according to: 1) Sleicher; 2) Rozentsvaig. d_*, m; μ_c, Pa·sec.

Fig. 2. Dependence of the size of the drops on the viscosity ratio of the phases according to: 1) Rozentsvaig, 2) Sleicher, 3) Laplace, 4) Kolmogorov–Hinze. d_*, D, m; μ_d, μ_c, Pa·sec.

In plotting Fig. 1, relations (5) and (10) were brought to the followin form by using identical transformations:

$$\log (d_*/A) = B \log \mu_c \,, \tag{11}$$

where the constants for Sleicher's formula are

$$A = AS = 38\,\sigma^{1.5}/(\rho_c u^{2.5})\,, \quad B = -0.5\,, \tag{12}$$

and for Rozentsvaig's formula are

$$A = AR = 56.246\,\sigma^{1.43}/(\rho_c^{1.02}\,\mu_d^{0.54}\,u^{2.45})\,, \quad B = 0.13\,. \tag{13}$$

In the upper half-plane of Fig. 1 experimental data a straight line that represents Sleicher's relation are given. In the lower half-plane the same are given, but for Rozentsvaig's relation. In both cases the experimental data are the same [3] (encircled).

It is evident that in the absence of the lower left experimental point, neither of the formulas can clearly be preferred. However, this point and the disposition of the straight lines relative to the regions occupied by the experimental data justify the choice of formula (5) for the case of water-in-oil emulsions.

The goodness-of-fit of formulas (5) and (10) to experimental data as regards the parameter μ_d/μ_c is evaluated in a similar way. Figure 2, in which $\alpha = 1.5$ and 1.43 and $\beta = 0.5$ and 0.41 for Sleicher's and Rozentsvaig's formula, respectively, confirms the earlier conclusion (see 1 and 2 in the upper part of Fig. 2). This figure also contains data (see 4) processed by Kolmogorov–Hinze's relation. It is seen that with in relation to Sleicher's experimental data formula (3) underestimates the value of the constant. The much larger area occupied by the experimental points in comparison to 1 and 2 testifies to the fact that formula (3) is much less adequate than formulas (5) and (10).

TABLE 1. Optimum Values of α and β in Formula (15)

α	β	$\alpha-\beta$	$\gamma=2\alpha-\beta$	Δ
0.9	0.1	0.8	1.7	0.586
1.0	0.2	0.8	1.8	0.245
1.1	0.3	0.8	1.9	0.216
1.3	0.4	0.9	2.2	0.208
1.2	0.4	0.8	2.0	0.209
1.4–1.3	0.5	0.8–0.9	2.1–2.3	0.207
1.7–1.5	0.6	0.8–0.9	2.2–2.4	0.206
1.6	0.7	0.9	2.5	0.207
1.7	0.8	0.9	2.6	0.241
1.7	0.9	0.8	2.5	0.283
1.8	1.0	0.8	2.6	0.332
a) 0.6	0.1	0.5	1.1	0.346
b) 1.5	0.5	1.0	2.5	0.209
c) 1.43	0.41	1.02	2.45	0.212
d) 1.0	0.25	0.75	1.75	0.236

Note: a) according to formula (3); b) (5); c) (10); d) (14).

Let us evaluate the size of the drops in a flow from the condition that a drop will break up if the averaged value of the pressure fluctuations, which is of the order of $\rho_c u_p^2$, is equal to or greater than the excess pressure in the drop. According to Laplace, this pressure is proportional to σ/d. Therefore, for $d = d_*$ we have the equality $\sigma/d_* = \text{const}\ \rho_c u_p^2$. According to Levich, for a circular tube $u_p^2 = \text{const}\ u^2/\text{Re}^{0.25}$. After substitution and transformations, we can easily obtain the following formula:

$$d_*/D = \text{const Re}^{0.25}\ \text{We}^{-1.0}\,. \tag{14}$$

Experimental data of Sleicher and relation (14) are given in Fig. 2 (denoted by 3). The solid line corresponds to (14), in which the constant estimated from Sleicher's data is equal to 14.25. In comparison with the cases considered earlier, relation (14) is only slightly inferior to formulas (5) and (10) in the spread of the experimental points. At the same time, it follows from relation (14) that $d_* = u^{-1.75}$, and thus this relation is intermediate between Kolmogorov–Hinze's and Sleicher's formula.

All the above relations are particular cases of the following general formula:

$$d_*/D = \text{const Re}^{\beta}\ \text{We}^{\alpha}\,. \tag{15}$$

Performing numerical investigations for various values of α and β, examined with a step of 0.1, pairs of their values were found for which the scatter in the experimental points (along the ordinate) turned out to be minimum. These are listed in Table 1. The table also contains values of the scatter determined from the formula

$$\Delta = \max\left\{\log\left(\frac{d_{*e}}{D}\frac{\text{We}^{\alpha}}{\text{Re}^{\beta}}\right)\right\} - \min\left\{\log\left(\frac{d_{*e}}{D}\frac{\text{We}^{\alpha}}{\text{Re}^{\beta}}\right)\right\}, \tag{16}$$

where d_{*e} is the size of the drops determined experimentally.

It is seen from the table that the difference between the optimum values of α and β remains virtually unchanged. From this fact and expression (15) it follows that the experimental data of Sleicher obey the dependence

$d_* \propto D^{0.2-0.1}$. In comparison, the analogous relations from formulas (3), (5), and (10) have exponents that are equal to 0.5, 0.0, and −0.02, respectively. This must be taken into account in using the formulas to calculate large-diameter pipelines.

According to Table 1, the minimum scatter corresponds to the values $\alpha = 1.4-1.5$ and $\beta = 0.6$. The results of an analysis of formulas (3), (5), (10), and (14) presented in the table show that Sleicher's formula is closest to the minimum scatter in the points.

The values of the exponent γ in the table for the dependence of d_* and $u^{-\gamma}$ show that they could lie entirely within the range of 1.9–2.5.

Summarizing, we can note that in all the analyses Sleicher's formula turned out to be preferable. Nevertheless, there are no grounds for rejecting the other relations until a sufficient body of experimental data is accumulated. The shortage in them is explained by the difficulties in carrying our "pure" experiments and the complexities in selecting phases that have viscosities, densities, and interphase stresses within the ranges needed for practice. As concerns the formulas, it should be noted that they were obtained on the basis of static consideration of the process of breakup, which is a dynamic process having a nonstationary character. This will manifest itself in a decrease in the most stable size of the drops in time. The effect is slight, but for large times of transport of an emulsion in a pipeline it leads to a noticeable decrease in the size of the drops. Unfortunately, this effect has received practically no study, because it cannot be investigated under laboratory conditions. And finally, in real emulsions a substantial effect on the breakup of the drops will be exerted by the inhomogeneity of the composition of the continuous phase and the presence in it of very small gas inclusions and solid particles. The latter, of sizes of the order of 1.0–0.01 μm with a sufficient numerical density, facilitate the process of breakup and thus reduce the actual value of d_*.

To calculate the maximum size of breakup-resistant drops in a pipeline we recommend use of the formula

$$d_*/D = 6.45\,\mathrm{Re}^{0.6}/\mathrm{We}^{1.4}.$$

NOTATION

d, D, diameter of the drops and the pipeline; μ, ρ, viscosity and density of the phases; σ, interphase tension; S, maximum velocity gradient; ε, energy of dissipation; u, flow velocity; λ, coefficient of hydraulic resistance; Re, We, Reynolds and Weber numbers; u_p, root-mean-square pulsational velocity. Subscripts: c and d, continuous and disperse phases.

REFERENCES

1. J. O. Hinze, AIChE J., **1**, No. 3, 289-295 (1955).
2. A. N. Kolmogorov, Dokl. Akad. Nauk SSSR, **66**, No. 5, 825-828 (1949).
3. C. A. Sleicher, AIChE J., **8**, No. 4, 471-477 (1962).
4. A. K. Rozentsvaig, Prikl. Mekh. Tekh. Fiz., No. 6, 71-78 (1981).